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The local integration of Leibniz algebras. (English, French summaries)

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This paper is dedicated to providing a local answer to the “coquecigrue” problem; namely, J.-L. Loday asked in [Enseign. Math. (2) **39** (1993), no. 3-4, 269–293; [MR1252069](#)] about the integration of Leibniz algebras. The problem consists in the definition of algebraic objects which correspond to Leibniz algebras in a similar way as groups correspond to Lie algebras.

A solution to the coquecigrue problem for split Leibniz algebras was given by M. K. Kinyon in [J. Lie Theory **17** (2007), no. 1, 99–114; [MR2286884](#)], where he showed that the algebraic structure which answers the problem is the structure of a digroup.

In the current article a complete answer to the coquecigrue problem is not given, in the sense that the author only constructs a local algebraic structure but not a global one; that is, to define an algebraic structure on a tangent space at a given point on a manifold, only an algebraic structure in a neighborhood of this point is needed. The author shows that a local answer to the problem is given by the pointed augmented local racks which are abelian extensions of a Lie group by an anti-symmetric module.

The core section of the paper recalls the link between (local) Lie racks and Leibniz algebras following [M. K. Kinyon, op. cit.], and then analyzes the passage from smooth $As(X)$ -modules to Leibniz representations and (local) Lie rack cohomology to Leibniz cohomology. A morphism from the (local) Lie rack cohomology of a rack X with coefficients in an $As(X)$ -module A^s (respectively A^a) to the Leibniz cohomology of the Leibniz algebra associated to X with coefficients in $\mathfrak{a}^s = T_0A$ (respectively $\mathfrak{a}^a$) is defined.

To integrate Leibniz algebras into local Lie racks, the author approaches the problem in a similar way as É. Cartan did for the Lie groups case in [C. R. Acad. Sci. Paris **190** (1930), 914–916, 1005–1007; JFM 56.0373.01]. Namely, for every Leibniz algebra, he considers the abelian extension by the left center and integrates it. This extension is characterized by a 2-cocycle and he constructs a local Lie rack 2-cocycle integrating it by an explicit construction similar to the Lie group case. This construction constitutes the main theorem in the paper. He remarks that the constructed 2-cocycle has more structure, namely the rack cocycle identity is induced by another one; this other identity allows one to endow the constructed local Lie rack with a structure of an augmented local Lie rack.

The section ends with an example of a Leibniz algebra integration in dimension 5.

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